Numerical Probability and Applications to Finance

Ecole Nationale des Ingénieurs de Tunis

April 30, 2015

9h00-9h30	:	Reception of the participants
9h30-10h00	:	Nicole EL KAROUI, University Paris 6 Robust estimation of abrupt change in Poisson intensity process
10h00-10h30	:	Gilles PAGÈS, University Paris 6 Multilevel Richardson Romberg Monte Carlo and Langevin Estimators
10h30-11h00	:	Damien LAMBERTON, UPEM On the Canadian approximation of the American put
11h00-11h30	:	*Coffee break*
11h30-12h00	:	Benjamin JOURDAIN, ENPC Strong convergence properties of the Ninomiya Victoir scheme and applications to multilevel Monte Carlo
12h00-12h30	:	Anis MATOUSSI, University du Maine TBA
12h30-14h30	:	*Lunch break and poster session*
14h30-15h00	:	Emmanuelle CLÉMENT, UPEM An application of the KMT construction to the pathwise weak error in the Euler approximation of the geometric brownian motion
15h00-15h30	:	Ahmed KEBAIER, University Paris 13 Importance Sampling and Statistical Romberg Method for Lévy Processes
15h30-16h00	:	*Coffee break*
16h00-16h30	:	Xiaolu TAN, University Paris-Dauphine Exact simulation of multi-dimensional stochastic differential equations
16h30-17h00	:	Mohamed MRAD, University Paris 13 Convergence rate of strong approximations of compound random maps

Abstracts

Speaker : E. Clément (LAMA, UPEM). Joint work with A. Gloter (LaMME-Université d'Évry).

Title : An application of the KMT construction to the pathwise weak error in the Euler approximation of the geometric Brownian motion

Abstract : It is well known that the strong error approximation, int the space of continuous paths equipped with the supremum norm, between a diffusion process, with smooth coefficients, and its Euler approximation with step $1/n$ is $O(n^{-1/2})$ and that the weak error estimation between the marginal laws, at the terminal time T, is $O(n^{-1})$. An analysis of the weak trajectorial error has been developed by Alfonsi, Jourdain and Kohatsu-Higa [1], through the study of the $p-$Wasserstein distance between the two processes. For a one-dimensional diffusion, they obtained an intermediate rate for the pathwise Wasserstein distance of order $n^{-2/3+\varepsilon}$. Using the Komlos, Major and Tusnady construction, we improve this bound in the case of the geometric Brownian motion and we obtain a rate of order $\log n/n$, which gets close to the marginal weak error.
[1] A.Alfonsi, B.Jourdain, and A.Kohatsu-Higa. Pathwise optimal transport bounds between a one-dimensional diffusion and its Euler scheme. *Ann. Appl. Probab.*, 24(3) : 1049-1080, 2014.

Speaker : B. Jourdain (CERMICS-ENPC). Joint work with A. Al Gerbi (CERMICS-ENPC) and E. Clément (LAMA, UPEM).

Title : Strong convergence properties of the Ninomiya Victoir scheme and applications to multilevel Monte Carlo

Abstract : We prove that the strong convergence rate of the Ninomiya-Victoir scheme is 1/2. The normalized error term converges stably to the solution of an affine SDE with a source term involving the commutators between the Brownian vector fields. When the Brownian vector fields commute, this limit vanishes and we show that the strong convergence rate improves to 1. We last show that averaging the order of integration of the Brownian fields leads to a scheme which couples with strong order 1 to the scheme recently proposed by Giles and Szpruch in order to achieve the optimal complexity in the multilevel Monte Carlo method.

Speaker : A. Kebaier (LAGA, UP13). Joint work with M. Ben Alaya (LAGA, UP13) and K. Hajji (LAGA, UP13).

Title : Importance Sampling and Statistical Romberg Method for Lévy Processes

Abstract : An important family of stochastic processes arising in many areas of applied probability is the class of Lévy processes. Generally, such processes are not simulatable especially for those with infinite activity. In practice, it is common to approximate them by truncating the jumps at some cut-off size ε, $\varepsilon \searrow 0$. This procedure leads us to consider a simulatable compound Poisson process. This paper first introduces, for this setting, the

statistical Romberg method to improve the complexity of the classical Monte Carlo one. Roughly speaking, we use many sample paths with a coarse cut-off $\varepsilon^b eta$, $\beta \in (0,1)$, and few additional sample paths with a fine cut-off ε. Central limit theorems of Lindeberg-Feller type for both Monte Carlo and statistical Romberg met hod for the inferred errors depending on the parameter ε are proved. This leads to an accurate description of the optimal choice of parameters with explicit limit variances. Afterwards, we propose a stochastic approximation method of finding the optimal measure change by Esscher transform for Lévy processes with Monte Carlo and statistical Romberg importance sampling variance reduction. Furthermore, we develop new adaptive Monte Carlo and statistical Romberg algorithms and prove the associated central limit theorems. Finally, numerical simulations are processed to illustrate the efficiency of the adaptive statistical Romberg method that reduces at the same time the variance and the computational effort associated to the effective computation of option prices when the underlying asset process follows an exponential pure jump CGMY model.

Speaker : M. Mrad (LAGA, UP13). Joint work with E. Gobet (CMAP-Ecole Polytechnique).

Title : Convergence rate of strong approximations of compound random maps

Abstract : We consider a random map $x \mapsto F(\omega, x)$ and a random variable $\Theta(\omega)$, and we denote by $F^N(\omega, x)$ and $\Theta^N(\omega)$ their approximations : We establish a strong convergence result, in $\mathbf{L}_p$-norms, of the compound approximation $F^N(\omega, \Theta^N(\omega))$ to the compound variable $F(\omega, \Theta(\omega))$, in terms of the approximations of F and Θ. Two applications of this result are then developed : Firstly, composition of two Stochastic Differential Equations through their initial conditions ; secondly, approximation of stochastic processes (possibly non semi-martingales) at random times (possibly non stopping times)

Speaker : G. Pagès (LPMA-UPMC). Joint work with V. Lemaire (LPMA-UPMC).

Title : Multilevel Richardson Romberg Monte Carlo and Langevin Estimators

Abstract : We propose and analyze a Multilevel Richardson-Romberg (*ML2R*) estimator which combines the higher order bias cancellation of the Multistep Richardson-Romberg extrapolation introduced in [Pagès 07] and the variance control resulting from the stratification in the Multilevel Monte Carlo (*MLMC*) method (see [Giles '08]). The *ML2R* estimator appears as a weighted version of the MLMC, with universal weights.

In standard frameworks like discretization schemes of diffusion processes, an assigned quadratic error epsilon can be obtained using the *ML2R* estimator with a global complexity of $\log(1/\varepsilon)\varepsilon^{(-2)}$ instead of $(\log(1/\varepsilon))^2\varepsilon^{(-2)}$ with the standard MLMC method, at least when the weak discretization error associated to (functionals of) the scheme can be expanded at any order in the step $\frac{T}{n}$ and the quadratic (strong) error behaves like $O(\sqrt{\frac{T}{n}}$. This is half-way between *MLMC* and a virtual unbiased simulation. More generally, the slower the quadratic strong error the goes to 0, the higher the complexity reduction is.

We analyze and compare these estimators on several numerical problems : option pricing (vanilla or exotic) using Monte Carlo simulation and the less classical Nested Monte Carlo simulation (see [Gordy & Juneja 2010]).

In a second step, we adapt similar ideas to Langevin Monte Carlo simulation for the recursive computation of invariant distributions of diffusions with applications to stationary stochastic volatility models.

Speaker : X. TAN (University Paris-Dauphine). Joint work with P. Henry-Labordère (Société Générale) and N. Touzi (CMAP-Ecole Polytechnique).

Title : Exact simulation of multi-dimensional stochastic differential equations

Abstract : We develop a weak exact simulation technique for a process X defined by a multi-dimensional stochastic differential equation (SDE). Namely, for a Lipschitz function g, we propose a simulation based approximation of the expectation $\mathbb{E}[g(Xt1, \cdots, Xtn)]$, which by-passes the discretization error. The main idea is to start instead from a well-chosen simulatable SDE whose coefficients are up-dated at independent exponential times. Such a simulatable process can be viewed as a regime-switching SDE, or as a branching diffusion process with one single living particle at all times. In order to compensate for the change of the coefficients of the SDE, our main representation result relies on the automatic differentiation technique induced by Elworthy's formula from Malliavin calculus, as exploited by Fournié and al. (1999) for the simulation of the Greeks in financial applications. Unlike the exact simulation algorithm of Beskos and Roberts (2005), our algorithm is suitable for the multi-dimensional case. Moreover, its implementation is a straightforward combination of the standard discretization techniques and the above mentioned automatic differentiation method.